Properties of Exponents and Logarithms

Exponents

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$ 2. $(a^m)^n = a^{mn}$ 3. $(ab)^m = a^m b^m$

4. $\dfrac{a^m}{a^n} = a^{m-n},\ a \neq 0$ 5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m},\ b \neq 0$ 6. $a^{-m} = \dfrac{1}{a^m},\ a \neq 0$

7. $a^{\frac{1}{n}} = \sqrt[n]{a}$ 8. $a^0 = 1,\ a \neq 0$ 9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

where m and n are integers in properties 7 and 9.

Logarithms

Definition: $y = \log_a x$ if and only if $x = a^y$, where $a > 0$.
In other words, logarithms are exponents.

Remarks:

- $\log x$ always refers to log base 10, i.e., $\log x = \log_{10} x$.
- $\ln x$ is called the natural logarithm and is used to represent $\log_e x$, where the irrational number $e \approx 2.71828$. Therefore, $\ln x = y$ if and only if $e^y = x$.
- Most calculators can directly compute logs base 10 and the natural log. For any other base it is necessary to use the change of base formula: $\log_b a = \dfrac{\ln a}{\ln b}$ or $\dfrac{\log_{10} a}{\log_{10} b}$.

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm	For logarithms base a
1. $\ln xy = \ln x + \ln y$	1. $\log_a xy = \log_a x + \log_a y$
2. $\ln \dfrac{x}{y} = \ln x - \ln y$	2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\ln x^y = y \cdot \ln x$	3. $\log_a x^y = y \cdot \log_a x$
4. $\ln e^x = x$	4. $\log_a a^x = x$
5. $e^{\ln x} = x$	5. $a^{\log_a x} = x$

Useful Identities for Logarithms

For the natural logarithm	For logarithms base a
1. $\ln e = 1$	1. $\log_a a = 1$, for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$, for all $a > 0$